Final Year M.Sc., Degree Examinations

September / October 2015

(Directorate of Distance Education)

MATHEMATICS

Paper – PM 10.08: DPB 540: NUMERICAL ANALYSIS

Time: 3hrs.] *[Max. Marks: 70/80*

Instructions to candidates:

i) ***Students who have attended 30 marks IA scheme will have to answer for total 70 Marks.***

ii) ***Students who have attended 20 marks IA scheme will have to answer for total 80 Marks.***

iii) ***Answer any FIVE questions for both 70-80 marks scheme and Question No. (9) in Section – B is compulsory for 80 marks.***

SECTION – A

1. a) Derive the Newton-Raphson scheme to obtain the roots of an equation $f(x)=0$. use it to find an approximate root of the equation $x^3-5x+1=0$.

   b) Describe Bairstow's method to extract a quadratic factor of the form x^2+px+q from a polynomial of degree *n*. (7 + 7)

2. a) Find the solution of

$$\begin{aligned} 83x+11y-4z&=95\\ 7x+52y+13z&=104\\ 3x+3y+29z&=71 \end{aligned}$$

   by performing four iterations using any one of iteration matrix.

   b) Explain successive over relaxation method to solve the system $Ax=b$ (8 + 6)

3. a) Find all the eigen values and the corresponding eigen vectors of the matrix

$$A=\begin{pmatrix} 1 & \sqrt{2} & 2\\ \sqrt{2} & 3 & \sqrt{2}\\ 2 & \sqrt{2} & 1 \end{pmatrix}.$$

   b) Find all the eigen values of the following matrix using Given's method

$$A=\begin{pmatrix} 1 & 6 & 0\\ 6 & 2 & 1\\ 0 & 1 & 3 \end{pmatrix}.$$ (8 + 6)

Contd...........2

4. a) Discuss the convergence criteria of Hermite interpolation polynomial of degree $\leq 2n+1$.

   b) Evaluate $I = \int_0^1 \frac{2x}{1+x^4}\,dx$ using Gauss Legendre and Gauss Chebyshev integration formula. (6 + 8)

5. a) Derive Lagrange's interpolation formula for the given data points $(x_i,\ y_i),\ i = 1,\ 2, \ldots n$

   b) Obtain the least square approximation polynomial of degree one and two for $f(x) = \sqrt{x}$ on $[0,\ 1]$. (7 + 7)

6. a) Determine the cubic spline $S(x)$ for the interval $[2,\ 3]$ for the following tabulated values of x and y.

X	1	2	3	4	5
Y	10	17	36	73	134

   b) Derive cubic spline interpolation polynomial. (7 + 7)

7. a) Derive Runge-Kutta 2nd order method to find the numerical solution to an IVP $y' = f(x, y),\ x_0, = y_0$

   b) Use Adams predictor-corrector method to find $y(0,8)$ and $y(1,0)$ for an IVP, $y' = x^2 + y,\ \ y(0) = 1,$ choose $h = 0.2$. (7 + 7)

8. a) Solve the Laplace equation $u_{xx} + u_{yy} = 0$ by employing five point formulae which satisfies the following boundary conditions.

   $u(0,\ y) = 0 \quad u(x,\ 0) = 0$

   $u(x,\ 1) = 100x,\ \ u(1,\ y) = 100y$ Choose $h = k = 1$.

   b) Derive Crank – Nicolson implicit formula for solving parabolic partial differential equation $\frac{\delta u}{\delta t} = \frac{\delta^2 u}{\delta x^2}$. (7 + 7)

SECTION – B

9. a) Use Secant method to find the real roots of $2x^3 + 3x - 5 = 0$, perform four iterations.

   b) Evaluate an integral $\int_0^1 \frac{dx}{x^2 + 2x + 2}$ by dividing the given interval into equal 4, 6, 8 subintervals using Simpson's ⅓rd rule. (5 + 5)

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