Final Year M.Sc., Degree Examinations

September / October 2015

(Directorate of Distance Education)

MATHEMATICS

Paper – PM 10.07: DPB 530: MEASURE THEORY AND FUNCTIONAL ANALYSIS

Time: 3hrs.] *[Max. Marks: 70/80*

Instructions to candidates:

i) ***Students who have attended 30 marks IA scheme will have to answer for total 70 Marks.***

ii) ***Students who have attended 20 marks IA scheme will have to answer for total 80 Marks.***

iii) ***Answer any FIVE questions for both 70-80 marks scheme and Question No. (9) in Section – B is compulsory for 80 marks.***

SECTION – A

1. a) Define a Borel Set. For any singleton set $\{x\}$, prove that $m^*\{(x)\}=0$.

   b) Construct an uncountable set of measure zero.

   c) If $\{I_n\}$ is a finite covering of open intervals of $Q\cap[0,1]$, show that $\sum l(I_n)\geq 1$. Is this true if $\{I_n\}$ is infinite? (5 + 4 + 5)

2. a) State Littlewood's three principles and prove any one of them.

   b) State and prove Fatou's lemma. (8 + 6)

3. a) If $f\geq 0$ and measurable, show that $\exists$ a sequence $\{\phi_n\}$ of simple functions such that $\phi_n\uparrow f$.

   b) Let $f:[a,b]\to R'$ be increasing. Show f' exists a.e., and that $\int_a^b f'\leq f(b)-f(a)$.

   (7 + 7)

4. a) Find the Dini Derivatives of $f(x)=\begin{cases} 0 & x=0 \\ x\sin\frac{1}{x} & x\neq 0\end{cases}$ at $x=0$.

   b) Define a function of bounded variation on $[a,b]$. With usual notations show that $T=N+P$ and $f(b)-f(a)=P-N$.

   c) Let *f* be absolutely continuous on $[a,b]$. Show that *f* is of bounded variation on $[a,b]$. (4 + 5 + 5)

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5. a) Define a complete metric space. Prove that $1_p, 1 \le p < \infty$ is a complete metric space. Do 1_∞ is complete? Justify.

   b) State and prove Banach Fixed point theorem. (8 + 6)

6. a) Show that there is no $f : R^1 \to R^1$ continuous only at rationals.

   b) State and prove Lebesgue Covering lemma. (6 + 8)

7. a) Prove that the set of all continuous linear operators of a normed linear space into a Banach space is itself a Banach space.

   b) Show that any two normed linear spaces with the same finite dimension are topologically isomorphic. (8 + 6)

8. a) State Hanh Banach theorem. Let M be a closed linear subspace of a normed linear space X and $x_0 \notin M$. If $d = d(x_0, M)$, then show that there exists a functional $f_0 \in X^*$ such that $f_0(M) = 0$ and $f_0(x_0) = 1$ and $\|f_0\| = \dfrac{1}{d}$.

   b) Prove that the space $l_p, 1 \le p < \infty$ is reflexive. Is the space l_1 reflexive? Justify. (8 + 6)

SECTION – B

9. If *f* is a real continuous function defined on a closed and bounded interval $[a, b]$ and if $\varepsilon > 0$, prove that there exists a polynomial p such that $|f(t) - p(t)| < \varepsilon$ for $a \le t < b$. (10)

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