Q.P. Code – 56913

M.Sc. (Final) Degree Examination

OCTOBER/NOVEMBER 2014

(Directorate of Distance Education)

Mathematics

(DPB 530) Paper PM 10.07 – MEASURE THEORY AND FUNCTIONAL ANALYSIS

Time : 3 Hours] *[Max. Marks : 70/80*

Instructions to Candidates :

1) *Students who have attended **30** marks **I-A** scheme will have to answer for total of **70** marks.*

2) *Students who have attended **20** marks **I-A** scheme will have to answer for total of **80** marks.*

3) *Answer any **FIVE** questions from Section-**A**. Each question carry **14** marks for both 70-80 marks scheme and Question No. **9** in Section-**B** is **compulsory** for **80**-marks scheme.*

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SECTION – A

1. (a) If $\{I_n\}$ is a finite covering of $Q \cap [0, 1]$ by open intervals, show $\sum 1(I_n) \geq 1$.

   (b) Define outer measure. Show that a countable set has outer measure zero. **7 + 7**

2. (a) Define a measurable function. If f and g are measurable then show that $f+g$ and fg are measurable.

   (b) Let f be a bounded function defined on a measurable set E of finite measure. Prove that $\inf_{f \leq \varphi} \int_E \varphi(x)dx = \sup_{f \geq \phi} \int_E \phi(x)dx$ for all simple functions defined on E if and only if f is measurable. **8 + 6**

3. (a) State and prove Fatou's lemma.

   (b) If f is an integrable function on $[a, b)$ and $F(x) = f(a) + \int_a^x f(t)dt$ then show that $F'(x) = f(x)$ for almost all $x \in [a, b]$. **7 + 7**

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4. (a) Find the Dini Derivatives of $f(x)=\begin{cases}0 & x=0\\ x\sin\frac{1}{x} & x\neq 0\end{cases}$.

(b) If f is absolutely continuous on $[a, b]$, show that f is Lipschitz on $[a, b]$. Is the converse true? **7 + 7**

5. (a) Define a separable space. Prove that 1_p $(1\leq p\leq\infty)$ is separable. Is l_∞ separable? Justify.

(b) If X is complete and $\{U_n\}_{n=1}^{\infty}$ is a sequence of dense open sets in X. Show $\bigcap_{n=1}^{\infty} U_n$ is dense in X.

6. (a) State and prove Cantor's intersection theorem.

(b) Define bounded and totally bounded metric space. Prove that every totally bounded metric space is bounded. Is the converse hold true? Justify.

(c) Show that a complete totally bounded metric space is compact. **5 + 5 + 4**

7. (a) Show that a normed linear space is finite dimensional if and only if the closed unit ball is compact.

(b) State Hahn Banach theorem. If M is closed linear subspace of a normed linear space X and $x_0 \notin X$, then show that there exists a functional $f_0 \in X^*$ such that $f_0(M)=0$ and $f_0(x_0)\neq 0$. **7 + 7**

8. State and prove open mapping theorem. Deduce closed graph theorem. **14**

SECTION – B

9. (a) Let $f: R' \to R'$ be measurable and $h(x)=\begin{cases}1 & f(x) \text{ rational}\\ 0 & f(x) \text{ irrational}\end{cases}$.

Show h is measurable.

(b) Show Q_1 is of 1st category and $R'-Q$ is of 2nd category. **6 + 4**
