Final Year M.Sc. Degree Examinations
December 2017
(Directorate of Distance Education)
MATHEMATICS
Paper – PM 10.05: DPB 510: COMPLEX ANALYSIS

Time: 3 hrs]   *[Max. Marks: 70/80*

Instructions to candidates:

1. *Students who have attended 30 marks IA scheme will have to answer for total 70 marks.*
2. *Students who have attended 20 marks IA scheme will have to answer for total of 80 marks.*
3. *Answer any FIVE questions from Section – A. Each question carries 14 marks for both 70 – 80 marks scheme and question No. 9 in Section – B is compulsory for 80 marks scheme.*

PART – A

1. a) Prove that $\left|\dfrac{a-b}{a-\bar{a}b}\right| < 1$, if $|a| < 1$ and $|b| < 1$.

    b) Obtain the spherical representation of the extended complex plane.

    c) Show that Z and Z^1 corresponds to diametrically opposite points on the Riemann sphere, if and only if $Z\,Z^{-1} = -1$. $(4 + 6 + 4)$

2. a) Show that inside the circle of convergence, power series represents an analytic function $f(z)$ which is infinitely differentiable.

    b) State and prove Abel's limit theorem.

    c) If $f(z) = u + iv$ is analytic and $u - v = e^x (\cos y - \sin y)$. Find $f(z)$ in terms of Z.
                                                                          $(5 + 5 + 4)$

3. a) Prove that product of any two bilinear transformation is again bilinear.

    b) Prove that the cross ratio (Z_1, Z_2, Z_3, Z_4) is real if and only if four points lies on the circle of a straight line.

    c) Find the bilinear transformation which carries $\{0, i, -i\}$ into $\{1, -1, 0\}$. $(4 + 6 + 4)$

4. a) State and prove Cauchy theorem for rectangle.

    b) State and prove Liouville theorem. Deduce fundamental theorem of algebra.

c) Verify Cauchy's integral theorem for the function $f(z) = z^2$ defined in the region bounded by the triangle with vertices $(0, 0)$, $(3, 0)$ and $(3, 1)$. $(5 + 5 + 4)$

5. a) State and prove maximum modulus principle and use it to derive Schwartz's lemma.

    b) Define removable, isolated and essential singularity. Give an example for each show that an isolated singularity of 'f' is removable iff $\lim_{z \to a}(z - a) f(z) = 0$.

    c) Expand $\log(1 + z)$ in a Taylors series about $z = 0$ and determine the region of convergence for the resulting series. $(5 + 5 + 4)$

6. a) Show that the successive derivatives of an analytic function at any point can never satisfy $\left| f^n(z) \right| > n! n^n$.

    b) Prove that if $f(z)$ is meromorphic inside a closed contour C and has no zeros on C then $\dfrac{1}{2\pi i} \int_C \dfrac{f'(z)}{f(z)} dz = N - P$, where, N is the number of zeros and P is the number of poles.

    c) Prove that the co-efficient in the Laurentz expansion of $f(z) = \sin\left(z + \dfrac{1}{z}\right)$ power series of Z are givne by the formula, $a_n = \dfrac{1}{2\pi} \int_C^{2\pi} \cos n\theta \sin(2\cos\theta) d\theta$ for $n = 1, 2$.

    $(4 + 6 + 4)$

7. a) State Laurent's theorem. Classify singularities using Laurent's theorem.

    b) Evaluate the following

    i) $\int_{-\infty}^{\infty} \dfrac{dx}{1 + x^4}$ ii) $\int_0^{\infty} \dfrac{\cos ax}{(a^2 + x^2)^2} dx$ $a > 0$ $(5 + 9)$

8. a) State and prove Weierstrass theorem.

    b) Derive Poisson's formula for a harmonic function.

    c) Show that $\sin \pi z = \pi z \prod_{n=1}^{\infty} \dfrac{(1 - z^2)}{n^2}$ $(5 + 5 + 4)$

PART – B

9. a) Show that $f(z) = \sin z$ is analytic and $f'(z) = \cos z$.

    b) Expand $f(z) = \dfrac{1}{(z + 1)(z + 3)}$ in Laurent's series valid for the regions

    i) $1 < |z| < 3$ ii) $0 < |z + 1| < 2$ $(6 + 4)$

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