Q.P. Code – 56921

First Year M.Sc. Degree Examination, OCTOBER/NOVEMBER 2016

(Directorate of Distance Education)

Physics

(DPA 510) MATHEMATICAL METHODS AND CLASSICAL MECHANICS

Time : 3 Hours] *[Max. Marks : 75/85*

Instructions to Candidates :

1) *Answer any **FIVE** questions from Parts A, B and C without omitting any Part.*
2) *Part **D** is **compulsory** for those who appear for paper with maximum marks **85**.*

PART – A

1. (a) State Cauchy's integral theorem and using it obtain Cauchy's integral formula.

   (b) Define an analytic function and examine the analyticity of $f(z) = z^2$.

   (c) Obtain the residues of the function $f(z) = \dfrac{2z+1}{(z-1)^2(z+2)}$ at $z = 1$. **7 + 5 + 3**

2. (a) Obtain the series Legendre of the Legendre differential equation $\left(1 - x^2\right)y'' - 2xy' + n(n+1)y = 0$.

   (b) Arrive at general solution of Helmholtz equation in spherical polar coordinates using separation of variable method. **8 + 7**

3. (a) Evaluate the vector identities : (i) $\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{A}\right)$ (ii) $\vec{\nabla}\times\vec{\nabla}\Phi$ where Φ, $\vec{A}$ are respectively scalar and vector functions.

   (b) Obtain the expression for Laplace operator ∇^2 in cylindrical polar coordinates.

   (c) State Gauss divergence theorem. **6 + 7 + 2**

PART – B

4. (a) State Cayley Hamilton theorem and use it to find the inverse of the matrix $\begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$.

   (b) Diagonalize the following matrix :

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

6 + 9

5. (a) Write down the transformation rule for a mixed tensor of rank (2, 3).

   (b) State quotient law for tensors and illustrate with an example.

   (c) Define (i) Contravariant and covariant vectors (ii) Contraction of indices in a tensor. **3 + 6 + 6**

6. (a) State and prove the convolution theorem for Fourier transforms.

   (b) Find the inverse Laplace transform of the function $f(s) = \frac{4}{(s^2+4)(s^2)}$.

   (c) If $F(w)$ is the Fourier transform of $f(x)$, then show that the Fourier transform $f(-x)$ is $F(-w)$. **7 + 5 + 3**

PART – C

7. (a) Obtain the principle of conservation of energy for a system of particles in a conservative force field.

   (b) Derive Lagrange's equation of motion from Hamilton's least action principle. **7 + 8**

8. (a) Obtain Kepler's laws of motion from a study of the central field motion of the sun and planet.

   (b) Derive Hamilton's equations of motion. **8 + 7**

9. (a) What are Poisson brackets? List their properties.

   (b) Express the canonical equations of Hamilton in terms of Poisson brackets.

   (c) Discuss the circumstances in which Hamiltonian represents total energy of a system. **6 + 4 + 5**

PART – D

10. Answer any **TWO** of the following : **2 × 5 = 10**

   (a) If $H_n(x)$ is Hermite polynomial, show that $H_{n+1}(x) = 2x\,H_n(x) - 2nH_{n-1}(x)$.

   (b) If $\phi(\vec{r})$ is a scalar function and $\dot{A}(\vec{r})$ is a vector function, show that

   $\vec{\nabla} \times (\phi \dot{A}) = \vec{\nabla}\phi \times \dot{A} + \phi(\vec{\nabla} \times \dot{A})$.

   (c) State the number of degrees of freedom for a one-dimensional simple pendulum and set up its Lagrangian in terms of generalized co-ordinates.
