Q.P. Code – 56904

First Year M.Sc. Degree Examination

SEPTEMBER/OCTOBER 2013

(Directorate of Distance Education)

Mathematics

(DPA 540) Paper – DIFFERENTIAL EQUATIONS

Time : 3 Hours] *[Max. Marks : 70/80*

Instructions to Candidates :

1) *Students who have attended* ***30*** *marks* ***I-A*** *scheme will have to answer for total of* ***70*** *marks.*

2) *Students who have attended* ***20*** *marks* ***I-A*** *scheme will have to answer for total of* ***80*** *marks.*

3) *Answer any* ***FIVE*** *questions from Section-****A****. Each question carry* ***14*** *marks for both 70-80 marks scheme and Question No.* ***9*** *in Section-****B*** *is* ***compulsory*** *for* ***80****-marks scheme.*

SECTION – A

1. (a) Prove that the solutions ϕ_1 and ϕ_2 of $y''+a_1y'+a_2y=0$ are linearly independent on an interval I iff $w(\phi_1, \phi_2)(x) \neq 0$ for all x in I.

(b) Consider two functions $\phi_1 = x^3$ and $\phi_2 = x^2|x|$, show that

(i) ϕ_1 and ϕ_2 are linearly independent on I

(ii) Find $w(\phi_1, \phi_2)(x)$

(iii) Do (i) and (ii) contradicts (a) on I? If so why? **7 + 7**

2. (a) Describe the method of variation of parameters to find the solution of the equation $y''+a_1y'+a_2y=b(x)$.

(b) State and prove Stum's comparison theorem. **7 + 7**

3. (a) State and prove the orthogonal property for Legendre differential equation.

(b) Find the solution of Hypergeometric equation

$x(1-x)y''+(c-(a+b+1)x)y'-ab\,y=0$ about $x=0$. **7 + 7**

4. (a) Obtain the solution of the Bessel's equation $x^2y'' + xy' + (x^2 - \alpha^2)y = 0$ about $x = 0$.

   (b) Prove that

   (i) $J_{-3/2}(x) = \sqrt{\dfrac{2}{\pi x}}\left(\dfrac{-\cos x}{x} - \sin x\right)$

   (ii) $J_{3/2}(x) = \sqrt{\dfrac{2}{\pi x}}\left(\dfrac{3\sin x}{x^2} - \dfrac{3\cos x}{x} - \sin x\right)$. **8 + 6**

5. (a) Find the Inverse Laplace transform of

   (i) $\dfrac{S}{(S^2 - 1)(S^2 - 4)}$

   (ii) $\log(S^2 + 1)$.

   (b) Solve the equation $y'' - 7y' + 6y = \sin x$, $y(0) = 0$, $y'(0) = 1$ using Laplace transform method. **6 + 8**

6. (a) State and prove Picard's existence and uniqueness theorem for an IVP $y' = f(x, y)$, $y(x_0) = y_0$.

   (b) Solve an IVP $y' = y(x-1)$, $y(0) = 0$, using Picards method. **7 + 7**

7. (a) Find the complete solution of the equation $(y + xz)p + (x + yz)q = z^2 - 1$ which passes through the curve $x = t$, $y = 1$, $z = t^2$.

   (b) Using Charpits method find the solution of the non-linear equation $z^2 = pqxy$. **7 + 7**

8. (a) Employ modified variable separable method to find the solution of $u_t = \alpha^2 u_{xx} + f(x, t)$, $0 \le x \le 1$, $t \ge 0$, when subjected to the boundary conditions $u(0, t) = u(1, t) = 0$ and initial conditions $u(x, 0) = f(x)$.

   (b) Find the solution of Dirichlets problem $u_{rr} + \dfrac{1}{r}u_r + \dfrac{1}{r^2}u_{\theta\theta} = 0$ when subjected to the boundary condition $u(R, \theta) = g(\theta)$, $0 \le \theta \le 2\pi$, $0 \le r \le R$ in the form of Poisson's integral formulae. **7 + 7**

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SECTION – B

9. (a) Find the eigen values and eigen functions for the equation $x'' + \lambda x = 0$, $x'(0) = 0,\ x'(\pi) = 0$.

(b) Classify and reduce the following equation into its canonical form, given $y^2 u_{xx} = x^2 u_{yy}$. **5 + 5**