M.Sc., (Previous), Degree Examination

August / September 2009

Directorate of Distance Education

(Freshers)

Mathematics

Paper : PM 10.03: Analysis - II

Time : 3 Hours Max. Marks : 80

Note : Answer any FIVE full questions.

1. a. If the series $\sum a_n$, $\sum b_n$, $\sum c_n$ converges to A, B, C and $C_n = a_0 b_n + \text{--------} + a_n b_0$, then show that $C = AB$. **06**

   b. Prove that the Cauchy product of two absolutely convergent series converges absolutely. **05**

   c. Show that with example that the product of two convergent series need not converges. **05**

2. a. Let $\sum a_n$ be a convergent series of real numbers but not absolutely and $-\infty \leq \alpha \leq \beta \leq \infty$. Then show that there exists a rearrangement $\sum \bar{a}_n$ with partial sums S_n such that

$$\lim_{n \to \infty} \sup \bar{S}_n = \beta, \quad \lim_{n \to \infty} \inf \bar{S}_n = \alpha$$ **10**

   b. Prove that the convergence of $\sum a_n$, implies the convergence of

$$\sum \sqrt{\frac{a_n}{n}} \text{ if } a_n \geq o.$$ **06**

3. a. Give an example to show that limit of the integral need not be equal to the integral of the limit even if both are finite.

   b. State and prove the weierstrass theorem for uniform convergence.

   c. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

4. a. Show that there exits a real continuous function on the real line which is nowhere differentiable with example. **08**

   b. Suppose $\{f_n\}$ is a sequence of differentiable functions on [a,b] and such that $\{f_n(x_0)\}$ converges for some point x_0 on [a,b]. If $\{f'_n\}$ converges uniformly on [a,b], then show that $\{f_n\} \to f$ on [a,b] uniformly and

$$\lim_{n \to \infty} f'_n(x) = f'(x) \ , \ x \in [a,b]$$ **08**

5. a. Define the Trigonometric function C(x) and S (x). Show that
   i) The function E is periodic with period $2\pi i$
   ii) The function C and S are periodic with period 2π.
   iii) If z is a complex number with $|z| = 1$, there is a unique t in $[0, 2\pi]$ such that $E(it) = z$. **10**

   b. Let $f(x) = \begin{cases} e^{-1/x^2} & (x \neq 0) \\ 0 & (x=0) \end{cases}$. Prove that f has derivatives of all orders at x=o and that $f^{(n)}(o) = 0$ for n= 1,2,3...... **06**

6. a. If f(x) and g(x) be two positive functions such that $f(x) \leq g(x)$, $x \in [a,b]$, then show that
   i) $\int_a^b f\,dx$ converges if $\int_a^b g\,dx$ converges
   ii) $\int_a^b g\,dx$ diverges if $\int_a^b f(x)$ diverges. **08**

   b. Define Beta and Gamma functions. Prove that Lagendre's duplicate formula. **08**

7. a. Suppose f maps an openset $E \subset \mathbb{R}^n$ in to $\mathbb{R}^m$ and f is differentiable at a point $x \in E$. Then show that the partial derivatives $(D_j f_i)(x)$ exist and

$$f'(x)\, e_j = \sum_{i=1}^{m} (D_j f_i)(x)\, u_i, \quad (1 \leq j \leq n).$$

10

   b. If $u = f(x,y)$ is a homogeneous function of degree n, then show that $x \frac{\delta u}{\delta x} + y \frac{\delta u}{\delta y} = nu$. **06**

8. State and prove the inverse function theorem. **16**

* * *