DPA – 520 (Math)

M.Sc. Mathematics (Previous) Examination, July /August 2011
(Directorate of Distance Education)
Paper : PM 10-02 : ANALYSIS – I

Time : 3 Hours Max. Marks : 70/80

Instructions : *1) Students who have attended* ***30*** *Marks IA Scheme will have to answer for total of* ***70*** *marks.*

2) Students who have attended ***20*** *Marks IA Scheme will have to answer for total of* ***80*** *Marks.*

3) Answer ***any FIVE*** *Questions from Section – A.* ***Each*** *question carries* ***14*** *marks (for both* ***70/80*** *Marks Schemes) and Q. No.* ***9*** *in Section –* ***B*** *is* ***compulsory*** *for* ***80*** *Marks Scheme.*

SECTION – A

1. a) Let $x, y \in R'$, $x > 0$. Show that there is a positive integer n such that nx > y. **8**

   b) Show that (x, y) contains infinitely many irrationals. **6**

2. a) If $\{E_n\}$ is a sequence of countable sets, show that $\bigcup_{n=1}^{\infty} E_n$ is countable. **7**

   b) Show that [0, 1] is uncountable. **7**

3. a) State and prove the Heine-Borel theorem. **7**

   b) Show that R' is connected. **7**

4. a) Let X and Y be metric spaces, $f : X \to Y$ be continuous, then prove that f is uniformly continuous. **5**

   b) Show that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$. Further show that e is irrational. **5**

   c) Prove that $\sum \frac{1}{n^p}$ is convergent if p > 1 and diverges if p < 1. **4**

P.T.O.

5. a) State and prove the generalized mean value theorem. **8**

b) Let $f : R' \to R'$ be differentiable, $f \neq 0$ and f (x+y) = f (x) f (y) $(x, y \in R')$. Show that f (x) = e^{Ax}, A is constant. **6**

6. a) Let f be bounded on [a, b], α increasing on [a, b]. Show that $f \in R(\alpha)$ on [a, b] iff $\exists$ a partition p of [a, b] $\ni S_p - s_p < \in$ $[\forall \in > 0]$. **8**

b) If |f| $\in$ R (α) on [a, b], does f $\in$ R (α) on [a, b] ? Prove your claim. **6**

7. a) Let f $\in$ R (α) on [a, b], $m \leq f \leq M$, ϕ continuous on [m, M]. Show that $\phi \circ f \in R(\alpha)$ on [a, b]. **9**

b) If $f_1, f_2 \in R(\alpha)$ on [a, b], show $f_1 \circ f_2 \in R(\alpha)$ on [a, b]. **5**

8. a) Show that a continuous function on [a, b] need not be of bounded variation on [a, b]. **7**

b) On [0, 1], let $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & 0 < x \leq 1 \\ 0, & x = 0 \end{cases}$, then show that f $\in$ BV [0, 1]. **7**

SECTION – B

9. a) If $f : X \to Y$ is continuous, where X is compact then prove that f (X) is compact. **5**

b) Show that log (1+ x^2) is uniformly continuous on R'. **5**