

Q.P. Code – 15631

Sixth Semester B.Sc. Degree Examinations, OCTOBER/NOVEMBER 2018

(Semester Scheme) (New Syllabus – 2014-15 onwards)

(SSF 530) Paper VII – MATHEMATICS

Time : 3 Hours]

[Max. Marks : 80

Note : Answer **ALL** questions.

I. Answer any TEN of the following :

10 × 2 = 20 marks

1. In a vector space $V(F)$, prove that $C\alpha = \bar{0} \Rightarrow$ either $C = 0$ or $\alpha = \bar{0}$.
2. Prove that the subset $W = \{(x, y, z) / \sqrt{2}x = \sqrt{3}y\}$ of the vector space $V_3(R)$ is a subspace of $V_3(R)$.
3. For what value of 'K', the vectors $(-4, -3, -2)$, $(-1, K, 1)$ and $(2, 3, 4)$ are linearly dependent.
4. If $T: V \rightarrow W$ is a linear transformation, then show that
 $T(0) = 0^1$ when $0 =$ zero vector of V and $0^1 =$ zero vector of W
5. Find the matrix of the linear transformation,
 $T: V_2(R) \rightarrow V_3(R)$ defined by $T(x, y) = (x - 2y, 2x, x - 3y)$
6. In an Euclidean vector space, prove that $|\alpha - \beta| \leq |\alpha| + |\beta|$
7. Prove that the inter section of two convex sets is also a convex set.
8. Draw the graph and find the solution set for the inequality $-1 \leq y \leq 2$ in R .
9. Define Degenerate solution of LPP.
10. If $P = \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ is a partition of $[0, 1]$, find
 $U(p, f)$ and $L(p, f)$ for the function of $f(x) = x$.
11. State Darboux theorem.
12. Prove that the lower Reimann integral cannot exceed the upper Reimann integral.

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II. Answer any THREE of the following :

3 × 5 = 15 marks

1. Prove that the intersection of two subspaces of a vector space is again a subspace and give an example to show "The Union of two subspaces of a vector space is need not be a subspace".
2. Express the polynomial $V = 5x^2 - 9x - 5$ as a linear combination of the polynomials $V_1 = 4x^2 + x + 2$, $V_2 = 3x^2 - x - 1$, $V_3 = 5x^2 - 2x + 3$.
3. In $V_3(Z_3)$ how many vectors are spanned by $(2, 1, 1)$ and $(1, 2, 2)$.
4. Prove that the set $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ of non zero vectors is linearly dependent iff some vector α_i is a linear combination of its preceding vectors.
5. Find the basis and dimesion of the subspace spanned by $S = \{(1, 1, 1), (2, 1, 2), (1, 0, 1), (5, 3, 5)\}$ in $V_3(R)$.

III. Answer any THREE of the following :

3 × 5 = 15 marks

1. Show that the transformation $T: V_2(R) \rightarrow V_2(R)$ defined by $T(x, y) = (x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)$ is linear.
2. Given $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{pmatrix}$. Find the linear transformation $T: V_3(R) \rightarrow V_2(R)$ relative to the bases $B_1 = \{(1, 1, 1), (1, 2, 3), (1, 0, 0)\}$ & $B_2 = \{(1, 1), (1, -1)\}$.
3. If $T: V \rightarrow W$ is a linear transformations, then prove that T is one - one iff $N(T) = \{0\}$ where 0 is zero vector of V. <https://www.kuvempuonline.com>
4. State and prove Rank - Nullity theorem.
5. Apply Gram - Schmidt process to orthonormalize the basis $\{(1, 0, 0), (1, 1, 1), (1, 2, 3)\}$.

IV. Answer any THREE of the following :

3 × 5 = 15 marks

1. Show that the set $S = \{(x_1, x_2) | 9x_1^2 - 4x_2^2 \leq 36\}$ is a convex set
2. Find the basic and basic feasible solution for the system,
 $x_1 - x_2 - 2x_3 = 9, \quad 3x_1 - 2x_2 - 5x_3 = 22$

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3. Two food stuff A and B have three vitamins V_1 , V_2 and V_3 as follows.

| Food Stuff | V_1 (mg) | V_2 (mg) | V_3 (mg) |
|------------|------------|------------|------------|
| A | 1 | 100 | 10 |
| B | 1 | 10 | 100 |

Minimum daily requirement of these vitamins are 1 mg, 50 mg and 10 mg respectively. If the cost of food stuff A is Rs 2 and that of B is Rs 3, find the minimum cost of diet that would supply the body at least minimum requirements of each vitamin by graphical method.

4. Max; $Z = 2x - 3y$

Sub to $x - y \leq 30$

$x - y \geq 0$

$y \geq 3$

$0 \leq x \leq 20$

$0 \leq y \leq 12$ by using graphical method.

5. Max; $f = 4x - 2y - z$

Sub to $x - y - z \leq 3$

$2x + 2y - z \leq 4$

$x - y \leq 0$

$x, y, z \geq 0$ by using simplex method.

V. Answer any THREE of the following :

3 × 5 = 15 marks

1. If P^* is the refinement of P then prove that $U(P^*, f) \leq U(P, f)$

2. Show that $f(x) = 3x + 1$ is R-integrable over $[1, 2]$ and hence deduce $\int_1^2 f(x) dx = \frac{11}{2}$

3. Prove that every continuous function is R-integrable.

4. If $f(x) = e^x$, compute $\int_0^1 f(x) dx$ by using integration as limit of sum.

5. Show that $f(x) = (-1)^{r-1}$ where $\frac{1}{r-1} \leq x \leq \frac{1}{r}$, $r = 1, 2, \dots$ is R-integrable over $[0, 1]$ and $\int_0^1 f(x) dx = \log 4 - 1$.