Q.P. Code – 15531

Fifth Semester B.Sc. Degree Examinations, APRIL/MAY 2018

(Semester Scheme) (Before 2014-15 – Old Syllabus)

(SSE 530) Paper V – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 80*

***Note** : Answer **ALL** questions.*

I. Answer any TEN of the following : 10 × 2 = 20 marks

1. In a vector space V over a field F, show that $c_1\alpha = c_2\alpha$ and $\alpha \neq 0 \Rightarrow c_1 = c_2$.

2. Let $V = R^3$, the vector space of all ordered triplets of real numbers, over the field of real numbers. Show that the subset $W = \{(x, 0, 0)/x \in R\}$ is a subspace of R^3.

3. Express the vector (3, 5, 2) as a Linear combination of the vectors (1, 0, 1), (2, 3, 0), (0, 0, 1) of $V_3(R)$.

4. Test the following set of vectors for linear dependence {(1, 0, 1), (0, 2, 2), (3, 7, 1)}.

5. Find the linear transformation $f : R^2 \to R^2$ such that $f(1, 0) = (1, 1)$ and $f(0, 1) = (-1, 2)$.

6. Prove that $|\alpha + \beta| \leq |\alpha| + |\beta|$ in any Euclidean space.

7. Evaluate $\int_C (3x - 2y)dx + (y + 2z)dy - x^2 dz$, where C is the curve $x = t$, $y = 2t^2$ and $z = 3t^3$ and $0 \leq t \leq 1$.

8. Evaluate $\int_0^3 \int_0^1 xy(x + y)\,dy\,dx$.

9. Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} \cdot dx\,dy\,dz$.

10. Evaluate $\dfrac{\Gamma 3 \cdot \Gamma 3/2}{\Gamma 9/2}$.

11. Evaluate $\beta(9, 7)$.

12. Show that $\int_0^\infty \dfrac{x^6(1 - x^8)}{(1 + x)^{22}}dx = 0$.

II. Answer any THREE of the following : **$3 \times 5 = 15$ marks**

1. If V is a vector space over a field F, then prove that

   (a) $C \cdot 0 = 0, \ \forall \ C \in F$

   (b) $0 \cdot \alpha = 0, \ \forall \ \alpha \in V$

   (c) $(-C) \cdot \alpha = -(C\alpha) = C \cdot (-\alpha), \ \forall \ C \in F, \ \alpha \in V$

2. Prove that an ordered set $\{\alpha_1, \alpha_2, \cdots \alpha_n\}$ of non-zero n vectors of a vector space $V[F]$ with $\alpha_1 \neq 0$, is linearly dependent iff one of the vectors say $\alpha_K (2 \leq K \leq n)$ is a linear combination of its preceeding ones.

3. In a n-dimensional vector space V, prove that

   (a) any $(n+1)$ elements of V are linearly dependent

   (b) no set of $(n-1)$ elements can span V.

4. How many vectors are spanned by (2, 1, 1) and (1, 2, 2) in $V_3(Z_3)$? Find them.

5. Determine the basis and dimension of the subspace spanned by the vectors (2, 4, 2), (1, –1, 0), (1, 2, 1) and (0, 3, 1).

III. Answer any THREE of the following : **$3 \times 5 = 15$ marks**

1. If $T : V_2(R) \to V_2(R)$ is defined by $T(x, y) = (3x + 2y, 3x - 4y)$ verify whether T is a linear transformation.

2. If $T : V \to W$ is a Linear transformation then prove that $T(0) = 0'$ and $T(-\alpha) = -T(\alpha)$, $\forall \alpha \in V$, where $0'$ is the zero vector of W.

3. Find the matrix of the Linear transformation $T : V_3(R) \to V_2(R)$ defined by $T(x, y, z) = (2x + 3y, y + 2z)$ with respect to the standard bases.

4. If $T : V_3(R) \to V_2(R)$ is defined by $T(x, y, z) = [y - x, y - z]$, find the range, null space and verify Rank-Nullity theorem.

5. If $\alpha = (3, 4, 5, 0)$ and $\beta = (1, 3, 0, 2)$ then find the angle between them.

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IV. Answer any THREE of the following : 3 × 5 = 15 marks

1. Evaluate $\int_C (3x+y)dx + (2y-x)dy$, along the curve $y = x^2 + 1$ from (0, 1) to (3, 10).

2. Show that

   $\int_C 2xy\,dx + (x^2 + 2yz)dy + (y^2+1)dz$ is independent of the path joining the points from origin to (1, 1, 1).

3. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dx\,dy}{1-x^2+y^2}$

4. Evaluate $\iint_R xy(x+y)dx\,dy$ where R is the region bounded by the parabola $y^2 = x$ and the line $y = x$.

5. Find the surface area of the sphere of radius a by using double integration.

V. Answer any THREE of the following : 3 × 5 = 15 marks

1. Evaluate $\iiint_R \left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x}\right) dx\,dy\,dz$, where R is the region $!< x < 2, 1 < y < 2$ and $1 < z < 2$.

2. Find the volume of the tetrahedron formed by the plane $x = 0, y = 0, z = 0$ and $6x + 4y + 3z = 12$.

3. Prove that $\beta(m, n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma(m+n)}$, $m, n > 0$.

4. Prove that $\int_0^{\pi/2} \sin^m\theta \cdot \cos^n\theta \cdot d\theta = \frac{1}{2} \cdot \frac{\Gamma\left(\frac{m+1}{2}\right) \cdot \Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{m+n+2}{2}\right)}$ and hence evaluate

   $\int_0^{\pi/2} \sin^3\theta \cdot \cos^5\theta \cdot d\theta$.

5. Evaluate $\int_0^2 (4-x^2)^{3/2} \cdot dx$

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