Q.P. Code – 15422

Fourth Semester B.Sc. Degree Examination, APRIL/MAY 2018

(Semester Scheme – Before 2014-15 Syllabus)

(SSD 530) Paper IV – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 80*

Note : *Answer **ALL** questions.*

I. Answer any TEN of the following : 10 × 2 = 20 marks

1. Define commutative Ring.

2. In a ring R show that $a \cdot 0 = 0 \cdot a = 0 \ \forall a \in R$.

3. Define sub-ring of a ring.

4. Show that Z is not an ideal of the ring $(Q, +, \cdot)$.

5. Show that the only idempotent elements of an integral domain are 0 and 1.

6. Find all the units of Z_8.

7. Evaluate $\lim_{n \to \infty} \left(\sqrt{n+1} - \sqrt{n}\right)$.

8. Examine the behaviour of the sequence whose nth term is given by $S_n = n \cdot \sin\left[\frac{\pi}{n}\right]$.

9. Define a bounded sequence and give an example.

10. Test the convergence of the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots$

11. Test the absolute convergence of the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + - + \ldots\ldots$

12. Test the convergence of the series $\sum \left[\frac{n}{n+1}\right]^{n^2}$

II. Answer any THREE of the following : **3 × 5 = 15 marks**

1. Show that the intersection of any two sub rings of a ring R is again a sub ring of R.

2. Show that every finite integral domain is a field.

3. Prove that a ring R has no zero divisors if and only if the cancellation laws hold in R.

4. Prove that the set $S = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}; a, b \in Z \right\}$ is a right ideal of the ring R of 2×2 matrices over Z. Show that it is not left ideal.

5. Find all the principal ideals of the ring $R = \{0, 1, 2, 3, 4, 5\}$ with respect to $+_6$ and $\times_6$.

III. Answer any THREE of the following : **3 × 5 = 15 marks**

1. If f is a homomorphism of a ring R onto a ring R' with Kernel K, then prove that $f(R)$ is homomorphic to factor ring R/K.

2. If $f ; R \to R'$ is a homomorphism from the ring R into R', then show that

   (a) $f(0) = 0'$ where $0, 0'$ are zeros of R and R'

   (b) $f(-a) = -f(a) \; \forall a \in R$

3. Let $N(\alpha) = a^2 - 3b^2$ where $\alpha = a + b\sqrt{3} \in z\left[\sqrt{3}\right]$ then prove that

   (a) $N(\alpha \cdot \beta) = N(\alpha) \cdot N(\beta) \; \forall \alpha, \beta \in z\left[\sqrt{3}\right]$

   (b) α is a unit iff $N(\alpha) = \pm 1$

4. Find the GCD of $x^{18} - 1$ and $x^{33} - 1$.

5. Test for rational roots of the polynomial $3x^3 + x^2 + x - 2$.

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IV. Answer any THREE of the following : **3 × 5 = 15 marks**

1. Show that every convergent sequence is bounded.

2. If $\{x_n\}$ and $\{y_n\}$ be two convergent sequences and let $\lim\limits_{n\to\infty} x_n = l$ and $\lim\limits_{n\to\infty} y_n = m$ then show that $\lim\limits_{n\to\infty} \left\{\dfrac{x_n}{y_n}\right\} = \dfrac{l}{m}$, where $m \neq 0$.

3. Show that the sequence $x_n = \dfrac{3n+4}{2n+1}$ is monotonic decreasing and bounded and show that $\lim\limits_{n\to\infty} x_n = \dfrac{3}{2}$.

4. Test the convergence of the sequence whose nth term is given by (i) $\left(\dfrac{n+1}{n-1}\right)^n$, (ii) $n\left[\log(n+1) - \log n\right]$.

5. Show that the sequence $\{S_n\}$ where $S_1 = 1$ and $S_n = \sqrt{2 + S_{n-1}}$, $\forall\ n \geq 2$ is convergent and converges to 2.

V. Answer any THREE of the following : **3 × 5 = 15 marks**

1. Discuss the convergence of the series $\dfrac{1}{1\cdot4} + \dfrac{1}{2\cdot5} + \dfrac{1}{3\cdot6} + \ldots$.

2. Test the convergence of the series $\dfrac{1}{3} + \dfrac{1\cdot2}{3\cdot5} + \dfrac{1\cdot2\cdot3}{3\cdot5\cdot7} + \ldots$.

3. State and prove Cauchy's root test.

4. Test the convergence of the series $x + \dfrac{1}{2}\dfrac{x^3}{3} + \dfrac{1\cdot3}{2\cdot4}\dfrac{x^5}{5} + \dfrac{1\cdot3\cdot5}{2\cdot4\cdot6}\dfrac{x^7}{7} + \ldots$.

5. Sum to infinity the series $1 + \dfrac{1}{5} + \dfrac{1\cdot3}{5\cdot10} + \dfrac{1\cdot3\cdot5}{5\cdot10\cdot15} + \ldots$.
