

DSC 231

Third Year B.Sc., Degree Examination
Directorate of Distance Education

July / August 2011

MATHEMATICS - IV

Time: 3 hrs.]

[Max.Marks: 90]

Note :- Answer any six of the following.

PART - A

1. (a) (i) Evaluate $\int_C y dx - x dy - z^2 dz$ where C is the curve $x = \sin t$,
 $y = \cos t$, $z = t^2$, $0 \leq t \leq 1$ 2 Marks
(ii) Evaluate $\int_0^1 \int_0^2 (x + y) dx dy$ 2 Marks
- (b) Show that $\int_C \frac{x^2 dy - y^2 dx}{x^{5/3} + y^{5/3}} = \frac{3\pi}{16} a^{4/3}$ where C is the quarter of the asteroid
 $x = a \cos^3 t$, $y = a \sin^3 t$ from the point $(a, 0)$ to the point $(0, a)$ 5 Marks
- (c) Evaluate $\iint_D (x^2 + y^2)^2 dx dy$ D is the region $r = \cos 2\theta$
 $-\pi/4 < \theta < \pi/4$ 6 Marks
2. (a) (i) Evaluate $\iint w \sin^{-1} z dz dw$ where $0 < z < 1$, $2 < w < 3$ 2 Marks
(ii) Evaluate $\int_0^2 \int_1^3 \int_1^2 xy^2 z dx dy dz$ 2 Marks
- (b) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dx dy dz$ 5 Marks
- (c) Find the volume common to the cylinder $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ 6 Marks
3. (a) (i) Show that $\beta(m, n) = \beta(n, m)$ 2 Marks
(ii) Show that $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta = \frac{r(3/4) r(1/4)}{2}$ 2 Marks
- (b) Prove that $\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right) = 2 \int_0^{\pi/2} \sin^p \theta \cos^q \theta d\theta$ 5 Marks
- (c) Prove that $\int_0^a y^4 \sqrt{(a^2 - y^2)} dy = \frac{\pi a^6}{32}$ 6 Marks
4. (a) (i) If $f: [a, b] \rightarrow \mathbb{R}$ is bounded function $P \in \mathcal{P}[a, b]$ then prove that
 $L(p, f) \leq U(p, f)$ 2 Marks
(ii) Prove that every continuous function is R – integrable 2 Marks
- (b) State and prove Darbrux theorem. 5 Marks
- (c) Show that $f(x) = x^2$ is R – integrable in $[0, 1]$ using lower and upper
Riemann integrals and $\int_0^1 x^2 dx = \frac{1}{3}$ 6 Marks

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PART - B

5. (a) (i) Find the complementary function of $y'' + (1 - \cot x)y' - y \cot x = \sin^2 x$ 2 Marks
(ii) Find the Wronskian W for the equation $y'' + y = \operatorname{cosec} x$ 2 Marks
- (b) Solve $y'' + (\tan x)y' + (4 \cos^2 x)y = 0$ by changing the independent variable 5 Marks
- (c) Solve $x^2 y'' - 2x(1+x)y' + 2(1+x)y = x^3$, $x > 0$ by changing the dependent variable given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = 1$ when $x = 1$. 6 Marks
6. (a) (i) Verify the condition of exactness of the equation $(1-x^2)y'' - 3xy' - y = 0$ 2 Marks
(ii) Write the complementary function for the cases $1 - P + Q = 0$ and $P + QX = 0$ 2 Marks
- (b) Solve $x^2y'' + xy' - y = 2x^2$, $x > 0$, given that $\frac{1}{x}$ is a part of complementary function. <https://www.kuvempuonline.com> 5 Marks
- (c) Solve $(D^2 + D)y = \operatorname{cosecx}$ by the method of variation of parameters. 6 Marks
7. (a) (i) Verify the condition of integrability of the function $yzdx - 2xzdy + (xy - zy^3)dz = 0$ 2 Marks
(ii) Form the partial differential equation by eliminating the arbitrary constants a and b in $(x - a)^2 + (y - b)^2 + z^2 = r^2$ 2 Marks
- (b) Solve $\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$ 5 Marks
- (c) Solve $P(mz - ny) + (nx - lz)q = (ly - mx)$ 6 Marks
8. (a) (i) Find the Fourier constants ' a_n ' and ' b_n ' with period $2L$. 2 Marks
(ii) If $f(x) = x - x^2$, $-1 < x < 1$ find a_0 2 Marks
- (b) Find the half range cosine series for $f(x) = \pi - x$ in the interval $(0, \pi)$ 5 Marks
- (c) Find the Fourier series of the function $f(x) = x^2$ in the interval $-\pi < x < \pi$ and hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ 6 Marks

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