Q.P. Code – 15222

Second Semester B.Sc. Degree Examinations, OCTOBER/NOVEMBER 2019

(Semester Scheme) (Old Syllabus – 2014 Onwards)

(SSB 530) Paper II – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 80*

Note *: Answer ALL questions.*

I. Answer any TEN of the following : 10 × 2 = 20 marks

1. If a/b and b/c, then prove that a/c.

2. Verify whether the number 223 is prime or not.

3. Find the remainder obtained when 2^{80} is divided by 5.

4. Find the ratio in which the line joining the points (3, 5, –4), (4, 2, 5) is divided by xy – plane.

5. Find the equation of the plane making intercepts (4, 5, 2) on the coordinate axes.

6. Find the equation of the plane passing through (–2, 1, 3) and parallel to the plane $5x - 3y + 7z + 3 = 0$.

7. Find the angle between the radius vector and the tangent for the curve $r^3 = a^2 \cos 2\theta$.

8. Find the pedal equation of the curve $r = a\theta$.

9. For the curve $y = a \log \sec\left(\frac{x}{a}\right)$, prove that $\frac{ds}{dx} = \sec\left(\frac{x}{a}\right)$.

10. If $Z = \log\sqrt{x^2 + y^2}$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 1$.

11. Let $G = \{1, -1, i, -i\}$ be a multiplicative group and $H = \{1, -1\}$ be a subgroup of G. Then prove that H is a normal subgroup of G.

12. Show that every quotient group of an abelian group is abelian.

 P.T.O.

II. Answer any THREE of the following : **3 × 5 = 15 marks**

1. Find the G.C.D. of (506, 1155) and express it in the form $506m + 1155n$.

2. Find the remainder when $7^{30} \times 3^{200}$ is divided by 5.

3. State and prove Fermat's theorem.

4. If n is even, prove that $n(n+1)(2n+1)$ is divisible by 24.

5. Solve $16x \equiv 8 \pmod{17}$.

III. Answer any THREE of the following : **3 × 5 = 15 marks**

1. If $P \equiv (2, 3)$ and $Q \equiv (4, 1)$, X lies on the line through P and Q. P lies between Q and X and $|\overrightarrow{PX}| = \frac{1}{4}|\overrightarrow{PQ}|$. Find the co-ordinates of X and express in terms of $\vec{P}$ and $\vec{Q}$.

2. Prove that in any triangle the line segment joining the mid points of any two sides is parallel to the third side and half of it.

3. Find the equation of the plane passing through the points $(1, 1, -1)$ and parallel to the line joining the points $A(2, -1, 0)$, $B(3, 1, 0)$ and $C(2, -2, 0)$, $D(3, -1, 0)$.

4. Determine the mutual position of the line l_1 and l_2 given by $l_1 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{3}$ and $l_2 : 3x + 2y - 4z + 7 = 0 = x - y + z - 8$.

5. The plane $y = 2$ intersects the sphere $x^2 + y^2 + z^2 - 4x - 6z - 7 = 0$ in a circle, find the centre and radius of this circle.

IV. Answer any THREE of the following : **3 × 5 = 15 marks**

1. Show that the curves $r = a\sec^2\left(\frac{\theta}{2}\right)$ and $r = a\,\text{cosec}^2\left(\frac{\theta}{2}\right)$ intersect orthogonally.

2. Obtain the pedal equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

3. Find the length of the perpendicular from the pole on to the tangent for the curve $r^2 = a^2 \cos 2\theta$.

4. Examine the function $z = 2x^3 + xy^2 + 5x^2 + y^2$ for maximum and minimum.

5. If $u = \tan^{-1}\left[\frac{x^3 + y^3}{x - y}\right]$ prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial u^2}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$.

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V. Answer any THREE of the following : 3 × 5 = 15 marks

1. Prove that a subgroup H of a group G is a normal subgroup of G if and only if every left coset of H in G is a right coset of H in G.

2. Prove that the centre Z of a group G is a normal subgroup of G.

3. Let $f : G \to G'$ be homomorphism form the group G in to the group G^1. Then prove that

   (a) $f[e] = e^1$, where e and e^1 are the identity elements of G and G^1 respectively.

   (b) $f[a^{-1}] = [f(a)]^{-1} \ \forall \ a \in G$.

4. Prove that, a homomorphism 'f' of a group G into a group G^1 is one-one if and only if $\ker f = \{e\}$.

5. Let G be a group of all real numbers under addition and G^1 be the group of non-zero complex numbers under multiplication. Show that $F : G \to G^1$ defined by $F(x) = e^{ix} \ \forall \ x \in G$ is a homomorphism. Find ker F. Is F an isomorphism?
