Q.P. Code – 15140

First Semester B.Sc. Degree Examination, OCTOBER/NOVEMBER 2019

(Semester Scheme – New Syllabus – 2018 Onwards)

Mathematics

(SSA 540) Paper BSM 1 – ALGEBRA – I AND CALCULUS - I

Time : 3 Hours] *[Max. Marks : 70*

***Note** : Answer all questions.*

I. Answer any FIVE of the following : **5 × 2 = 10**

1. If A is symmetric or Skew – symmetric, then prove that $AA^1=A^1A$.

2. Prove that every eigen vector of a matrix corresponds to one and only one eigen value of A.

3. Reduce the matrix $A=\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ 8 & -1 & -1 & 4 \end{bmatrix}$ to the echelon form.

4. Find the angle between the radius vector and the tangent for the curve

   $a\,\theta = (r^2 - a^2)^{1/2} - a\cos^{-1}\left(\frac{a}{r}\right)$

5. If $x = a\cos^3 t, y = a\sin^3 t$ find $\frac{ds}{dt}$.

6. Find the n^{th} derivative of a^{5x}.

7. Find fx, fy for $f(x,y) = \sin(x/y)$.

8. If $u = ax^2 + 2xy + by^2$, then verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

II. Answer any THREE of the following : **3 × 5 = 15**

1. Prove that the rank of the matrix is unaltered by interchanging any two rows of the matrix.

2. Find the rank of the matrix $A=\begin{bmatrix} 0 & 0 & -2 & 3 & 1 \\ 2 & 4 & 1 & 4 & 3 \\ 1 & 2 & -3 & 1 & 2 \\ 4 & 8 & 2 & 3 & 5 \end{bmatrix}$

3. Find the non-trivial solution of the system,

   $x_1+2x_2+2x_3 = 0;\ 2x_1+x_2+x_3=0;\ x_2+x_3=0.;3x_1+2x_2+2x_3=0$

P.T.O.

4. Test the following system for consistency and solve if it is consistent.
   $5x+3y+7z-4=0;\ 3x+26y+2z=9;\ 7x+2y+10z-5=0$

5. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

III. Answer any THREE of the following : 3 × 5 = 15

1. Diagonalise the matrix $\begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}$
2. State and prove Cayley – Hamilton theorem.
3. Find the inverse of the matrix A= $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$ using Cayley Hamilton theorem.
4. Prove that the angle between the radius vector and tangent at a point on the curve is $\tan\phi - r\frac{d\theta}{dr}$
5. Show that the curves $r=a\ sec^2\ {}^{\theta}/_{2}$ and $r=a\ cosec^2\ \frac{\theta}{2}$ intersect orthogonally.

IV. Answer any THREE of the following : 3 × 5 = 15

1. Obtain the pedal equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
2. Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point $\left(\frac{a}{4},\frac{a}{4}\right)$
3. Find the co-ordinates of the centre of curvature at (x,y) for the curve $y = a\cosh\left(\frac{x}{y}\right)$.
4. Find the n^{th} derivative of $e^{ax}\sin(bx+c)$.
5. Find the n^{th} derivative of $\frac{x+3}{(x-1)(x+2)}$

V. Answer any THREE of the following : 3 × 5 = 15

1. State and prove Leibnitz's theorem.
2. If $y=\sin[\log(x+1)^2]$ prove that $(x+1)^2 y_{n+2}+(2n+1)(x+1)y_{n+1}+(n^2+4)y_n=0$.
3. If $u=\sin^{-1}\left(\frac{x^3-y^3}{x-y}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2\tan u$.
4. Find the extreme value of $x^3+y^3-63x-63y+12xy=0$.
5. Show that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube.
