Q.P. Code – 15140

First Semester B.Sc. Degree Examination, OCTOBER/NOVEMBER 2018

(Semester Scheme – New Syllabus – 2018 onwards)

Mathematics

(SSA 540) Paper BSM 1 – ALGEBRA – I AND CALCULUS – I

Time : 3 Hours] *[Max. Marks : 70*

Note *: Answer ALL questions.*

I. Answer any FIVE of the following : **5 × 2 = 10 marks**

1. If A is a symmetric (skew - symmetric) matrix, then show that KA is symmetric (skew – symmetric) where K is any scalar.

2. Show that the matrices A and B are equivalent, where

$$A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 & -2 \\ 0 & 4 & -1 \\ 0 & 0 & 5 \end{bmatrix}.$$

3. If λ is an Eigen value of the matrix A then prove that λ^2 is an Eigen value of A^2.

4. Find the slope of the curve $r = e^{\theta}$ at $\theta = 0$.

5. For the curve $y = \log(\sec(x/a))$, show that $\frac{ds}{dx} = \sec(x/a)$.

6. Find the n^{th} derivative of $y = \frac{1}{3x+2}$

7. If $u = e^{x/y}$, then find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$

8. If $x = r\cos\theta$ $y = r\sin\theta$ then prove that $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 1$

II. Answer any THREE of the following : **3 × 5 = 15 marks**

1. Show that the multiplication of every element of any row by a non-zero constant does not alter the rank of the matrix.

2. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$ using elementary row operations.

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3. Solve completely the system of equations, $x+3y-2z = 0$, $2x-y+4z=0$ $x-11y+14z=0$.

4. Verify the system of equations $x+2y - z=3, 3x - y+2z=1$, $2x - 2y+3z = 2$ for consistency and hence solve if it is consistent.

5. Find the eigen values and the eigen vectors of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$.

III. Answer any THREE of the following : 3 × 5 = 15 marks

1. Diagonalise the matrix A= $\begin{pmatrix} 2 & 4 \\ 0 & 5 \end{pmatrix}$

2. Using Cayley – Hamilton theorem, find the adjoint of the matrix $\begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$

3. Find A^3 for the matrix A= $\begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$ by using Cayley – Hamilton theorem.

4. Show that the angle between the normal at any point $(r.\theta)$ on the curve $r^m=a^m \cos m\theta$ and the initial line is $(m+1)\theta$.

5. Show that the curves $r=a(1+\cos\theta)$ and $r=b(1-\cos\theta)$ intersect orthogonally.

IV. Answer any THREE of the following : 3 × 5 = 15 marks

1. Show that the pedal equation of the parabola $y^2 = 4a(x+a)$ is $p^2 = ar$.

2. Show that the radius of curvature for Cartesian curve $y = f(x)$ is $\rho = \dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3\,2}}{\dfrac{d^2y}{dx^2}}$.

3. Find the Co-ordinates of centre of curvature at the point $(at^2, 2at)$ for the curve $y^2 = 4ax$

4. Find the n^{th} derivative of $\dfrac{x}{2x^2-3x-2}$.

5. Find the n^{th} derivative of $y=(ax+b)^m$ where m is a positive integer and $n \le m$ and also obtain y_n when $m=-1$.

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V. Answer any THREE of the following : **3 × 5 = 15 marks**

1. State and prove Leibnitz theorem.

2. If $y=\left[\log\left(x+\sqrt{1+x^2}\right)\right]^2$ show that $(1+x^2)\, y_{n+2}+(2n+1)\, xy_{n+1}+ n^2y_n=0$

3. If $u=\log\sqrt{x^2+y^2+z^2}$ then prove that $(x^2+y^2+z^2)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)=1$

4. If $u=\sec^{-1}\left[\frac{x^3+y^3}{x+y}\right]$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2\cot u$.

5. Find the maximum and minimum values of the function $f(x,y)=x^3+3xy^2-3x^2-3y^2+4$.
